

ALGORITHM 229

ELEMENTARY FUNCTIONS BY CONTINUED
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Code None

Cat 20

```
procedure CONFRAC (x, n, parm, answer);
  integer parm, n; real x, answer;
```

comment This procedure utilizes a continued fraction which is equivalent to the diagonal of the Padé table for $\exp z$, with error in the computed convergent less than $x^{2n}/(2 \times 6^2 \times (10)^2 \times \dots \times (4n-2)^2(4n+2))$. This fraction was developed by J. C. Morelock, Note on Padé Table Approximation, Internal Note MIN-COMP-62-9, Marshall Space Flight Center, Huntsville, Alabama, 1962. For source reference see Nathaniel Macon, On the computation of exponential and hyperbolic functions using continued fractions, *J. ACM*, 2(1955), 262-266. The argument, x , is assumed to be less than $\pi/4$. For such x any desired level of accuracy is quickly computed for each function specified as follows:

```
parm := 1, answer := sin x  parm := 5, answer := sinh x
parm := 2, answer := cos x  parm := 6, answer := cosh x
parm := 3, answer := tan x  parm := 7, answer := tanh x
parm := 4, answer := exp x
```

The body of this procedure has been tested using extended ALGOL for the B-5000 Computer. It gave the following results:

```
x = 0.50  n = 1  parm = 1  answer = 0.47938 801530
x = 0.50  n = 2  parm = 1  answer = 0.47942 547125
x = 0.50  n = 3  parm = 1  answer = 0.47942 553854
x = 0.50  n = 4  parm = 1  answer = 0.47942 553860
x = 0.50  n = 1  parm = 2  answer = 0.87760 305992
x = 0.50  n = 2  parm = 2  answer = 0.87758 259869
x = 0.50  n = 3  parm = 2  answer = 0.87758 256193
x = 0.50  n = 4  parm = 2  answer = 0.87758 256189
x = 0.50  n = 1  parm = 3  answer = 0.54624 697337
x = 0.50  n = 2  parm = 3  answer = 0.54630 239019
x = 0.50  n = 3  parm = 3  answer = 0.54630 248974
x = 0.50  n = 4  parm = 3  answer = 0.54630 248985
x = 0.50  n = 1  parm = 4  answer = 1.64864 864865
x = 0.50  n = 2  parm = 4  answer = 1.64872 139973
x = 0.50  n = 3  parm = 4  answer = 1.64872 127057
x = 0.50  n = 4  parm = 4  answer = 1.64872 127070
x = 0.50  n = 1  parm = 5  answer = 0.52104 563580
x = 0.50  n = 2  parm = 5  answer = 0.52109 539374
x = 0.50  n = 3  parm = 5  answer = 0.52109 530541
x = 0.50  n = 4  parm = 5  answer = 0.52109 530549
x = 0.50  n = 1  parm = 6  answer = 1.12760 301285
x = 0.50  n = 2  parm = 6  answer = 1.12762 600598
x = 0.50  n = 3  parm = 6  answer = 1.12762 596516
x = 0.50  n = 4  parm = 6  answer = 1.12762 596521
x = 0.50  n = 1  parm = 7  answer = 0.46204 251473
x = 0.50  n = 2  parm = 7  answer = 0.46211 721881
x = 0.50  n = 3  parm = 7  answer = 0.46211 715720
x = 0.50  n = 4  parm = 7  answer = 0.46211 715726
```

The value of n selects the continued fraction convergent;

```
begin integer i, ndigit;
  real r, f;
  r := if parm ≤ 3 then -x ↑ 2 else x ↑ 2;
  f := 4 × n + 2;
  for i := n step -1 until 1 do f := 4 × i - 2 + r/f;
  ndigit := if parm ≤ 3 then parm + 1 else parm - 3;
  answer := if ndigit = 1 then (f+x)/(f-x)
    else if ndigit = 2 then 2 × x × f/((f ↑ 2) - r)
    else if ndigit = 3 then ((f ↑ 2)+r)/((f ↑ 2)-r)
    else if ndigit = 4 then 2 × x × f/((f ↑ 2)+r)
    else x;
end
```